

Name Answer Key

ID _____

Midterm 3 - 60 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. No cell phones. Do not use your own scratch paper.

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature _____

Suppose that you wish to predict light truck prices as a function of a few basic characteristics:

$$\log(\text{price}) = \beta_0 + \beta_1 \log(\text{weight}) + \beta_2 \log(\text{length}) + \beta_3 \log(\text{width}) + \beta_4 \log(\text{cyl}) + u$$

Here, *price* is measured in dollars, *weight* is measured in pounds, length and width are measured in inches, and *cyl* is the number of cylinders in the engine. The results from estimating this equation are below:

Source	SS	df	MS	
Model	90.4147706	4	22.6036926	Number of obs = 3297
Residual	148.927025	3292	.045239072	F(4, 3292) = 499.65
				Prob > F = 0.0000
				R-squared = 0.3778
				Adj R-squared = 0.3770
				Root MSE = .21269
Total	239.341795	3296	.072615836	

logprice	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
logweight	1.091928	.0322143	XX		
loglength	-.173286	.0357808	XX		
logwidth	-1.001126	.0807677	XX		
logcyl	.0428736	.025818	XX		
_cons	5.942897	.2774355	XX		

a.) Using the 95% confidence level, test whether the coefficient on *log(cyl)* is significantly different from zero. Please state your null and alternative hypotheses, and briefly interpret the result. (10 Points)

$H_0: \beta_4 = 0$ vs $H_A: \beta_4 \neq 0$
 $t_{stat} = \frac{0.429 - 0}{0.258} = 1.66$
 $t_{crit} = 1.96$

$|t_{stat}| < t_{crit} \Rightarrow$ Fail to reject the null.
 at 95% confidence, there is no significant relationship between the #cylinders and price.

b.) Suppose I claim that the elasticity of price with respect to width is not equal to -1. What is the probability that I'm wrong? (10 Points)

$$t_{stat} = \frac{-1.0011 - (-1)}{0.0808} = -0.0136$$

$$P_{value} = 2(1 - P(T < |t_{stat}|)) =$$

$$+6 = 2 \cdot (1 - 0.504) = \underline{0.992}$$

c.) Suppose I claim that $\beta_1 + \beta_3 = 0$. Please state a null and alternative hypotheses that can test this claim, and derive an equation that allows me to test the null against the alternative. Show your work!! (10 Points)

$$H_0: \theta = \beta_1 + \beta_3 = 0$$

$$(\theta - \beta_1) = \beta_3$$

$$+2 \quad H_A: \theta \neq 0$$

$$\Rightarrow \text{subs for } \beta_3$$

+2

$$\log(\text{price}) = \beta_0 + \beta_1 \log(\text{weight}) + \beta_2 \log(\text{length}) + \beta_3 \log(\text{width}) + \beta_4 \log(\text{cyl}) + u$$

$$= \beta_0 + \beta_1 \log(\text{weight}) + \beta_2 \log(\text{length}) + (\theta - \beta_1) \log(\text{width}) + \beta_4 \log(\text{cyl}) + u$$

$$\log(\text{price}) = \beta_0 + \beta_1 (\log(\text{weight}) - \log(\text{width})) + \beta_2 \log(\text{length}) + \theta \log(\text{width}) + \beta_4 \log(\text{cyl}) + u$$

+6

d.) I decide that I'm adding too many variables in predicting the vehicle price. Instead I estimate

$$\log(\text{price}) = \beta_0 + \beta_2 \log(\text{length}) + \beta_4 \log(\text{cyl}) + u$$

The results from estimating this equation are below:

Source	SS	df	MS	Number of obs = 3297		
Model	37.8161576	2	18.9080788	F(2, 3294) =	309.32	
Residual	201.66028	3294	.061127699	Prob > F =	0.0000	
				R-squared =	0.1579	
				Adj R-squared =	0.1574	
				Root MSE =	.24724	

logprice	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
loglength	.3153531	.0362388	XX		
logcyl	.4304159	.0246745	XX		
_cons	7.38903	.1782998	XX		

Is this model preferred to the model in 'a'? Please test this at the 95% level, stating your null and alternative hypotheses. Show your work!!! (10 Points)

$$F_{\text{stat}} = \frac{\frac{SSR_R - SSR_{ur}}{q}}{\frac{SSR_{ur}}{df_{ur}}} + b$$

$$= \frac{\frac{201.66 - 148.9}{2}}{\frac{148.9}{3292}} = 583.23$$

$H_0: \beta_1 = \beta_3 = 0$ +1
 $H_A: H_0 \text{ not true}$ +1

$$F_{\text{crit}} = 3$$

+1

$$F_{\text{stat}} > F_{\text{crit}} \Rightarrow$$

+1

Reject the null

e.) I'm having second thoughts about using logs, and instead estimate the following:

$$price = \beta_0 + \beta_1 weight + \beta_2 cyl + \beta_3 ltr + u$$

Where the additional variable *ltr* is the liters displacement by the engine. The results are below:

Source	SS	df	MS			
Model	3.2852e+10	3	1.0951e+10	Number of obs =	3298	
Residual	7.4300e+10	3294	22556107.4	F(3, 3294) =	485.49	
				Prob > F =	0.0000	
				R-squared =	0.3066	
				Adj R-squared =	0.3060	
				Root MSE =	4749.3	
price	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
weight	4.012499	.134814	XX			
cyl	447.3331	117.2983	XX			
ltr	-635.1207	114.4063	XX			
cons	3137.16	573.5268	XX			

Please construct a 90% confidence interval for the coefficient on *weight*. Please interpret this confidence interval. (10 Points)

$$\hat{\beta}_1 - se(\hat{\beta}_1) \cdot t_{crit} < \beta_1 < \hat{\beta}_1 + se(\hat{\beta}_1) \cdot t_{crit}$$

$$4.012 - 0.1348 \cdot 1.645 < \beta_1 < 4.012 + 0.1348 \cdot 1.645$$

$$\boxed{3.79 < \beta_1 < 4.234}$$

① with 90% confidence a 1 lbs increase in ~~price~~ ^{weight} has between a \$3.79 and \$4.23 increase in the price.

or
② ~~the~~ Weight has a ^{positive one} statistically significant effect on the wage.

+4

f.) Finally, after generating the new variables, I run the following regression:

$$price = \beta_0 + \beta_1(\text{weight} - 5000) + \beta_2(\text{cyl} - 8) + \beta_3(\text{ltr} - 5.4) + u$$

The results are below:

Source	SS	df	MS			
Model	3.2852e+10	3	1.0951e+10	Number of obs =	3298	
Residual	7.4300e+10	3294	22556107.4	F(3, 3294) =	485.49	
				Prob > F =	0.0000	
				R-squared =	0.3066	
				Adj R-squared =	0.3060	
Total	1.0715e+11	3297	32499913.2	Root MSE =	4749.3	

price	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
weight - 5000	4.012499	.134814	XX		
cyl - 8	447.3331	117.2983	XX		
ltr - 5.4	-635.1207	114.4063	XX		
_cons	23348.67	124.8494	XX		

Please construct a 95% confidence interval for the constant. Please interpret this confidence interval. (10 Points)

$$23,348.67 - 1.96(124.8494) < \beta_0 < 23,348.67 + 1.96 \cdot 124.8494$$

U

$$23,103.39 < \beta_0 < 23,592.61$$

+2

With 95% confidence, a vehicle with 8 cyl

5.4 ltr, and weight = 5000 will cost

between \$23,103.39 and \$23,592.61, +8

+0 if they say "an increase in the constant"